Operátory

$$|\psi\rangle \longrightarrow \alpha|a\rangle = \langle a|\psi\rangle |a\rangle$$

"sugestivní" přepsání

$$= |a\rangle\langle a|\psi\rangle \equiv \underbrace{|a\rangle\langle a|}_{\text{něco působící na } |\psi\rangle} |\psi\rangle$$

$\alpha|a\rangle \leftarrow$  $|\psi\rangle$

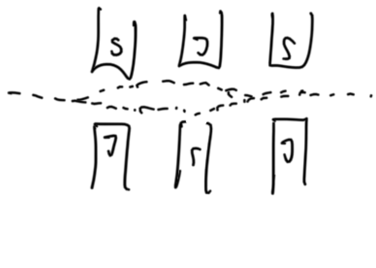
Filtr $\quad \hat{F}_a = |a\rangle\langle a|$

SG experiment

$$\hat{F}_\uparrow = |+z\rangle\langle +z| \quad ; \quad \hat{F}_\uparrow |\psi\rangle = \langle +z|\psi\rangle |+z\rangle$$

$$\hat{F}_\downarrow = |-z\rangle\langle -z|$$

Sjednocení svazků



$$\underbrace{\left(\hat{F}_\uparrow + \hat{F}_\downarrow\right)}_{\mathbb{1} \text{ jednotkový operátor}} |\psi\rangle = |\psi\rangle$$

Zkusme případ $|+x\rangle$

$$\left(\hat{F}_{\downarrow}+\hat{F}_{\uparrow}\right)|+x\rangle = \left(\hat{F}_{\uparrow}+\hat{F}_{\downarrow}\right)\frac{1}{\sqrt{2}}\left(|+z\rangle+|-z\rangle\right) =$$

$$= \frac{1}{\sqrt{2}}\left(|+z\rangle\underbrace{\langle +z|+z\rangle}_{1} + |+z\rangle\underbrace{\langle +z|-z\rangle}_{=0} + |-z\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(|+z\rangle+|-z\rangle\right) = |+x\rangle$$

obecně

$$|\psi\rangle = \alpha|+z\rangle+\beta|-z\rangle \quad ; \quad \left(\hat{F}_{\uparrow}+\hat{F}_{\downarrow}\right)|\psi\rangle = \alpha|+z\rangle+\beta|-z\rangle = |\psi\rangle$$

$$\boxed{\hat{F}_{\uparrow}+\hat{F}_{\downarrow} = \mathbb{1}}$$

$$\boxed{\hat{F}_{\uparrow}\hat{F}_{\downarrow}|\psi\rangle = 0}$$

$$|+z\rangle\overbrace{\langle +z|-z\rangle}^{=0}\langle -z|\psi\rangle$$

$$= 0$$

Relace úplnosti

$|n\rangle \qquad n = 1, \dots, N \qquad \dots$ N-rozměrný prostor

Hilbertův prostor

$$\boxed{\sum_n |n\rangle\langle n| = \mathbb{1}}$$

Příklad:

$$|1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \sum_n |n\rangle\langle n| = |1\rangle\langle 1| + |2\rangle\langle 2| + |3\rangle\langle 3|$$

$$|2\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad = \begin{pmatrix}1\\0\\0\end{pmatrix}\underbrace{1\ 0\ 0} + \begin{pmatrix}0\\1\\0\end{pmatrix}\underbrace{0\ 1\ 0} + \begin{pmatrix}0\\0\\1\end{pmatrix}\underbrace{0\ 0\ 1}$$

$$|3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$