Operátory spinu

$$\hat{S}_z = \frac{1}{2}|+z\rangle\langle+z| - \frac{1}{2}|-z\rangle\langle-z|$$

operátor komponenty spinu ve směru osy z

Obdobně pro osu x

$$\hat{S}_x = \frac{1}{2}|+x\rangle\langle+x| - \frac{1}{2}|-x\rangle\langle-x|$$

Co s osou y?

Víme, že

$$|\langle+z|+x\rangle|^2 = |\langle+z|-x\rangle|^2 = \frac{1}{2} \qquad \langle-z|+z\rangle = 0$$

$$|\langle-z|+x\rangle|^2 = |\langle-z|-x\rangle|^2 = \frac{1}{2} \qquad \langle-x|+x\rangle = 0$$

stejně musí být pro y

$$|\langle+z|+y\rangle|^2 = |\langle+z|-y\rangle|^2 = \frac{1}{2} \qquad \langle-y|+y\rangle = 0$$

...

navíc

$$|\langle+x|+y\rangle|^2 = |\langle+x|-y\rangle|^2 = \frac{1}{2}$$

$$|\langle-x|+y\rangle|^2 = |\langle-x|-y\rangle|^2 = \frac{1}{2}$$

$$|+z\rangle = \begin{pmatrix}1\\0\end{pmatrix}; \quad |-z\rangle = \begin{pmatrix}0\\1\end{pmatrix} \rightarrow \quad |+x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$|-x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

$$|+y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}; \quad |-y\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix}$$

Ověření:

$$\langle +y|-y\rangle = \frac{1}{2}\begin{pmatrix}1 & -i\end{pmatrix}\begin{pmatrix}1\\ -i\end{pmatrix} = \frac{1}{2}(1-1) = 0$$

$$\langle +x|+y\rangle = \frac{1}{2}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}1\\ i\end{pmatrix} = \frac{1}{2}(1+i)$$

$$|\langle +x|+y\rangle|^2 = \frac{1}{4}(1+1) = \frac{1}{2} \quad \dots \text{ atd.}$$

$$\Rightarrow \quad \hat{S}_y = \frac{1}{2}|+y\rangle\langle +y| - \frac{1}{2}|-y\rangle\langle -y|$$

Doposud vše reprezentováno v bázi vektorů $|+z\rangle$ a $|-z\rangle$

$$|+z\rangle = \begin{pmatrix}1\\ 0\end{pmatrix} \qquad |+x\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 1\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ 0\end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ 1\end{pmatrix}$$

$$|-z\rangle = \begin{pmatrix}0\\ 1\end{pmatrix} \qquad = \frac{1}{\sqrt{2}}|+z\rangle + \frac{1}{\sqrt{2}}|-z\rangle$$

Úloha:

Vyjádřete operátory $\hat{S}_z$, $\hat{S}_x$ a $\hat{S}_y$ pomocí matic v bázích $\{|+z\rangle, |-z\rangle\}$, $\{|+x\rangle, |-x\rangle\}$ a $\{|+y\rangle, |-y\rangle\}$

Zapište je v příslušných bázích pomocí tzv. Pauliho matic

$$\sigma_1 = \sigma_x = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

$$\sigma_2 = \sigma_y = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

$$\sigma_3 = \sigma_z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$\hat{S}_z = \frac{1}{2}\sigma_z$$