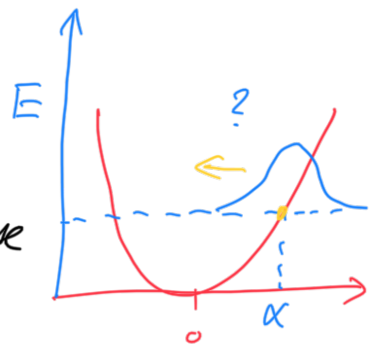
Časový vývoj koherentního stavu

$$\psi_0(q-\alpha) = e^{\frac{\alpha}{\sqrt{2}}(a^\dagger - a)}\,\psi_0(q)$$



$$\boxed{|\alpha\rangle} = e^{\frac{\alpha}{\sqrt{2}}(a^\dagger - a)}|0\rangle$$ — vývoj v čase

koherentní stav

$$|\alpha(t)\rangle = \; ? \; = U(t)|\alpha(0)\rangle$$

$$= e^{-\frac{i}{\hbar}Ht}|\alpha(0)\rangle = e^{-i\omega a^\dagger a t}\, e^{\frac{\alpha}{\sqrt{2}}(a^\dagger - a)}|0\rangle$$

$$e^X e^Y = e^Z \qquad Z = X + Y + \frac{1}{2}[X,Y] + \dots$$

$$[a^\dagger, a] = a^\dagger a - a a^\dagger = -1$$

$$e^{-\frac{1}{2}\frac{\alpha}{\sqrt{2}}}\, e^{\frac{\alpha}{\sqrt{2}}(a^\dagger - a) + \frac{1}{2}\frac{\alpha}{\sqrt{2}}} = \boxed{e^{-\frac{1}{2}\frac{\alpha}{\sqrt{2}}}\, e^{\frac{\alpha}{\sqrt{2}}a^\dagger}\, e^{-\frac{\alpha}{\sqrt{2}}a} = e^{\frac{\alpha}{\sqrt{2}}(a^\dagger - a)}}$$

$$|\alpha(0)\rangle = e^{-\frac{1}{2}\frac{\alpha}{\sqrt{2}}}\, e^{\frac{\alpha}{\sqrt{2}}a^\dagger}|0\rangle \qquad e^{\xi a}|0\rangle = \left(1 + \xi a + \frac{1}{2}\xi^2 a^2 \dots\right)|0\rangle = |0\rangle$$

úloha:

$$e^{-\frac{i}{\hbar}Ht}\, e^{\frac{\alpha}{\sqrt{2}}a^\dagger}|0\rangle$$

$$\boxed{e^{-\frac{i}{\hbar}\hat{H}t}\, a^\dagger} = \left(1 - \frac{i}{\hbar}\hbar\omega a^\dagger a t + \left(-\frac{i}{\hbar}\right)^2 (\hbar\omega)^2 a^\dagger a a^\dagger a t^2 + \dots\right) a^\dagger$$

$$= \sum_n (-i\omega)^n (a^\dagger a)^n a^\dagger t^n = \sum_n \frac{(-i\omega)^n}{n!} a^\dagger (a a^\dagger)^n t^n$$

$$= a^\dagger \sum_n \frac{(-i\omega)^n}{n!} (a^\dagger a + 1)^n = a^\dagger e^{-\frac{i}{\hbar}Ht - i\omega t}$$

$$e^{-\frac{i}{\hbar}\hat H t}\underbrace{a^\dagger a^\dagger a^\dagger \dots a^\dagger}_{n} = a^\dagger e^{-i\omega t} e^{-\frac{i}{\hbar}\hat H t}(a^\dagger)^{n-1} = \dots = \underbrace{(a^\dagger)^n e^{-in\omega t}}_{(a^\dagger e^{-i\omega t})^n} e^{-\frac{i}{\hbar}Ht}$$

$$e^{-\frac{i}{\hbar}\hat H t} e^{\frac{\alpha}{\sqrt2}a^\dagger} = e^{\frac{\alpha}{\sqrt2}a^\dagger e^{-i\omega t}} e^{-\frac{i}{\hbar}Ht} \Rightarrow e^{-\frac{i}{\hbar}Ht} e^{\frac{\alpha}{\sqrt2}a^\dagger}|0\rangle = e^{\frac{\alpha}{\sqrt2}e^{-i\omega t}a^\dagger}|0\rangle$$

$$\Rightarrow \boxed{|\alpha(t)\rangle = e^{+\frac{\alpha_0 e^{-i\omega t}}{\sqrt2}a^\dagger - \frac{\alpha_0}{\sqrt2}e^{i\omega t}a}|0\rangle}$$

$$\psi_0(q) \to \psi_0(q-\alpha)$$

$$|\alpha(t)\rangle = e^{\frac{\alpha_0}{\sqrt2}\cos\omega t(a^\dagger - a) - i\frac{\alpha_0}{\sqrt2}\sin\omega t(a^\dagger + a)}|0\rangle$$

$$= e^{-i\alpha_0\cos\omega t\,\hat p - i\alpha_0 \sin\omega t\,\hat q}|0\rangle$$

$e^X e^Y = e^Z;$

$Z = X + Y + \frac{1}{2}[X, Y]$

$$\approx e^{-i\alpha_0 \sin\omega t\, \hat{q}} \underbrace{e^{-i\alpha_0 \cos\omega t\, \hat{p}} |0\rangle}_{\text{posunutí o } q(t) = \alpha_0 \cos\omega t}$$

$$|\alpha_{q=\alpha_0 \cos\omega t}\rangle$$

v impulsové reprezentaci

$$|\alpha(t)\rangle = e^{-\alpha_0 \sin\omega t \frac{\partial}{\partial p}} |\alpha_{q=\alpha_0 \cos\omega t}\rangle$$

$$= |\alpha_{q=\alpha_0 \cos\omega t;\, p=\alpha_0 \sin\omega t}\rangle$$