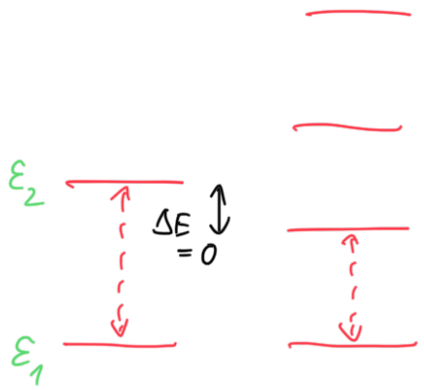
Role rezonance při přechodech mezi dvěma stavy



$$\vec{E} = \vec{E}_0 \cos\omega t$$

$$H_I = -\vec{\mu}\cdot\vec{E}(t) = -\begin{pmatrix} 0 & \vec{d} \\ \vec{d} & 0 \end{pmatrix}\vec{E}_0 \cos\omega t$$

$$= \begin{pmatrix} 0 & J\cos\omega t \\ J\cos\omega t & 0 \end{pmatrix} \quad ; J = -\vec{d}\cdot\vec{E}_0$$

$$H = \begin{pmatrix} \varepsilon_1 & J\cos\omega t \\ J\cos\omega t & \varepsilon_2 \end{pmatrix}$$

Schrödingerova rovnice

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -\frac{i}{\hbar} H(t)\,|\psi(t)\rangle$$

Interakční obraz

Stavový vektor v interakčním obrazu

$$H(t) = \underbrace{H_0}_{\begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{pmatrix}} + \underbrace{H_I(t)}_{\begin{pmatrix} 0 & J\cos\omega t \\ J\cos\omega t & 0 \end{pmatrix}}$$

$$U_0(t,t_0) = \exp\left(-\frac{i}{\hbar} H_0 (t-t_0)\right)$$

$$U_0(t_0,t) = \exp\left(+\frac{i}{\hbar} H_0 (t-t_0)\right)$$

$$|\psi^{(I)}(t)\rangle = U_0(t_0,t)\,|\psi(t)\rangle$$

Pohybová rovnice

$$\frac{\partial}{\partial t}|\psi^{(I)}(t)\rangle = \left(\frac{\partial}{\partial t} U_0(t_0,t)\right)|\psi(t)\rangle + U_0(t,t_0)\frac{\partial}{\partial t}|\psi(t)\rangle$$

$$= \cancel{\frac{i}{\hbar} H_0 \underbrace{U_0(t_0,t)|\psi(t)\rangle}_{|\psi^{(I)}(t)\rangle}} + U_0(t,t_0) \times \left(\cancel{-\frac{i}{\hbar} H_0 |\psi(t)\rangle} - \frac{i}{\hbar} H_I(t)|\psi(t)\rangle\right)$$

$$= -\frac{i}{\hbar} U_0(t,t_0) H_I(t) \underbrace{U_0(t,t_0) U_0(t_0,t)}_{\mathbb{1}} \overbrace{U_0(t_0,t)|\psi(t)\rangle}^{|\psi^{(I)}(t)\rangle}$$

$$\frac{\partial}{\partial t}|\psi^{(I)}(t)\rangle = -\frac{i}{\hbar} H_I^{(I)}(t)|\psi^{(I)}(t)\rangle$$

$$H_I^{(I)}(t) = U_0(t,t_0) H_I(t) U_0(t,t_0)$$

Dvě důležité vlastnosti interakčního obrazu

1) Obecné rovnice nebudou explicitně obsahovat H_0

2) V bázi, kde je H_0 diagonální se touto transformací nemění obsazení stavů

Dokažme 2)

$$U_0(t,t_0) = \sum_n e^{-\frac{i}{\hbar}E_n(t-t_0)}|n\rangle\langle n|$$

$$P_n^{(I)}(t) = \langle\psi^{(I)}(t)|n\rangle\langle n|\psi^{(I)}(t)\rangle = \langle\psi(t)|U_0(t,t_0)|n\rangle\langle n| \times U_0(t_0,t)|\psi(t)\rangle =$$

$$= \langle\psi(t)|\cancel{e^{-\frac{i}{\hbar}E_n(t-t_0)}}|n\rangle\langle n|\cancel{e^{\frac{i}{\hbar}E_n(t-t_0)}}|\psi(t)\rangle = P_n(t)$$

QED

$$\frac{\partial}{\partial t}|\psi^{(I)}(t)\rangle = -\frac{i}{\hbar} H_I^{(I)}(t)|\psi^{(I)}(t)\rangle$$

$$H_I^{(I)}(t) = U_0(t_0, t)\, H_I(t)\, U_0(t, t_0)$$

Jak vypadá $H_I^{(I)}(t)$?

$$H_I^{(I)}(t) = \begin{pmatrix} e^{\frac{i}{\hbar}\varepsilon_1(t-t_0)} & 0 \\ 0 & e^{\frac{i}{\hbar}\varepsilon_2(t-t_0)} \end{pmatrix} \begin{pmatrix} 0 & J\cos\omega t \\ J\cos\omega t & 0 \end{pmatrix} \begin{pmatrix} e^{-\frac{i}{\hbar}\varepsilon_1(t-t_0)} & 0 \\ 0 & e^{-\frac{i}{\hbar}\varepsilon_2(t-t_0)} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & J e^{-\frac{i}{\hbar}(\varepsilon_2-\varepsilon_1)(t-t_0)}\cos\omega t \\ J e^{\frac{i}{\hbar}(\varepsilon_2-\varepsilon_1)(t-t_0)}\cos\omega t & 0 \end{pmatrix}$$

$t_0 = 0$

$$\cos\omega t = \frac{1}{2}\left(e^{-i\omega t} + e^{i\omega t}\right) \quad ; \quad \omega_{21} = \frac{\varepsilon_2 - \varepsilon_1}{\hbar}$$

$$e^{i\omega_{21}t}\cos\omega t = \frac{1}{2}\left(e^{-i\underbrace{(\omega-\omega_{21})}_{\Delta\omega}t} + e^{i(\omega+\omega_{21})t}\right)$$

$|\psi^{(I)}(t)\rangle =$

$= a(t)|1\rangle + b(t)|2\rangle$

$$\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = -\frac{i}{\hbar}\begin{pmatrix} 0 & \frac{J}{2}\left(e^{-i\Delta\omega t} + e^{-i(\omega+\omega_{21})t}\right) \\ \frac{J}{2}\, c.c. & 0 \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\Rightarrow \frac{\partial}{\partial t} a(t) = -\frac{i}{\hbar}\frac{J}{2}\left(e^{-i\Delta\omega t} + e^{-i(\omega+\omega_{21})t}\right) b(t)$$

pro $J=0$ $\quad$ a i b konstantní

Předpokládejme $b(0) = b_0 = 1$

$$a(t) = \int_0^t d\tau \left(-\frac{i}{\hbar}\right)\frac{J}{2}\left(e^{-i\Delta\omega\tau} + e^{-i(\omega+\omega_{21})\tau}\right) b_0$$

$$= -\frac{i}{4}\frac{J}{2}\frac{1}{-i\Delta\omega}\left(1-e^{-i\Delta\omega t}\right)b_0 - \frac{i}{4}\frac{J}{2}\frac{1}{-i(\omega+\omega_{21})}\times\left(1-e^{-i(\omega+\omega_{21})t}\right)b_0$$

$\approx \frac{J}{\omega}$ ... malé

$J \ll \omega$

$\frac{J}{\Delta\omega}$ ~ velké pro $\Delta\omega \to 0$

$$\frac{1}{\cancel{-i\Delta\omega}}\left(\cancel{1}-\cancel{1}+\cancel{i\Delta\omega}\, t+\ldots\right)$$

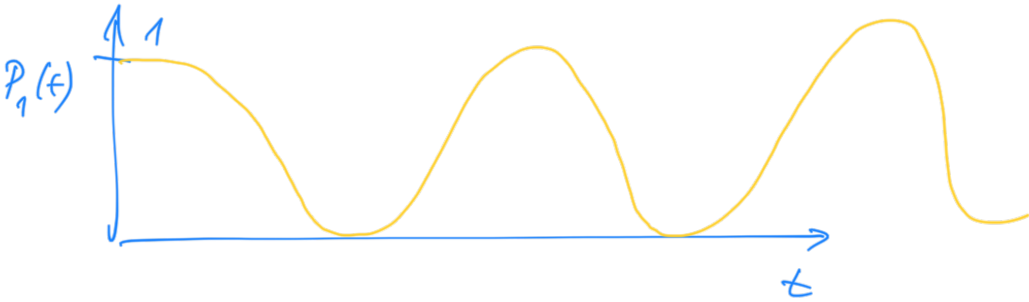
$$H_I^{(I)}(t) \longrightarrow \tilde{H}_I^{(I)}(t) = \begin{pmatrix} 0 & \frac{J}{2}e^{-i\Delta\omega t} \\ \frac{J}{2}e^{i\Delta\omega t} & 0 \end{pmatrix}$$

$\Delta\omega \to 0$ $\quad \tilde{H}_I^{(I)} = \begin{pmatrix} 0 & \frac{J}{2} \\ \frac{J}{2} & 0 \end{pmatrix}$

$E_1 = -\frac{J}{2}$; $E_2 = +\frac{J}{2}$; $\phi = \frac{\pi}{4}$

$$\boxed{P_1(t)} = P_g(t) = \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 + 2\left(\frac{1}{\sqrt{2}}\right)^4\cos(Jt)$$

$$= \boxed{\frac{1}{4}+\frac{1}{4}+\frac{1}{2}\cos(Jt) = \frac{1}{2}+\frac{1}{2}\cos(Jt)}$$



(Obecné) řešení problému s časově závislou vazbou

$$H = \begin{pmatrix} 0 & J \\ J & \varepsilon \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & \varepsilon \end{pmatrix}}_{H_0} + \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix}$$

$$H_I^{(I)} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega t} \end{pmatrix} \begin{pmatrix} 0 & J \\ J & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\omega t} \end{pmatrix} = \begin{pmatrix} 0 & Je^{-i\omega t} \\ Je^{i\omega t} & 0 \end{pmatrix}$$

Připomíná případ s časově proměnnou vazbou

$$\overline{H} = \begin{pmatrix} \varepsilon_1 & 0 \\ 0 & \varepsilon_2 \end{pmatrix} + \begin{pmatrix} 0 & J\cos\omega t \\ J\cos\omega t & 0 \end{pmatrix}$$

$$\overline{H}_I^{(I)}(t) = \begin{pmatrix} 0 & \frac{J}{2} e^{-i\delta\omega t} \\ \frac{J}{2} e^{i\delta\omega t} & 0 \end{pmatrix} \longrightarrow$$

lze transformovat do časově nezávislého hamiltoniánu

$$\begin{pmatrix} 0 & J \\ J & \varepsilon \end{pmatrix} \longleftrightarrow \begin{pmatrix} 0 & Je^{-i\omega t} \\ Je^{i\omega t} & 0 \end{pmatrix}$$

$$|\psi(t)\rangle \to |\psi^{(I)}(t)\rangle = U_0(t_0,t)|\psi(t)\rangle$$

tomu odpovídá

$$\begin{pmatrix} 0 & \frac{J}{2} \\ \frac{J}{2} & \hbar\delta\omega \end{pmatrix} \longleftrightarrow \begin{pmatrix} 0 & \frac{J}{2} e^{-i\delta\omega t} \\ \frac{J}{2} e^{i\delta\omega t} & 0 \end{pmatrix}$$

$$|\psi(t)\rangle$$

$$= U_0(t_0,t)|\psi^{(S)}(t)\rangle$$

$$|\psi^{(S)}(t)\rangle = U(t,t_0)\,|\psi(0)\rangle$$

$$U_0(t,t_0) = e^{-\frac{i}{\hbar}H_0(t-t_0)}$$

$$|\psi(t)\rangle = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\Delta\omega(t-t_0)} \end{pmatrix} U(t,t_0)|\psi(0)\rangle$$

$$H_0 = \begin{pmatrix} 0 & 0 \\ 0 & \hbar\Delta\omega \end{pmatrix}$$

nebudeme provádět žádné transformaci

nemění populace stavů

Tedy

$P_1(t)$ odpovídá případu $H = \begin{pmatrix} \varepsilon_1 & \frac{J}{2} \\ \frac{J}{2} & \varepsilon_2 \end{pmatrix}$

$$\Delta\varepsilon = \hbar\Delta\omega = \varepsilon_2 - \varepsilon_1$$

$$\boxed{\phi = \frac{1}{2}\arctan\left(\frac{J}{\hbar\Delta\omega}\right)}$$

$$P_1^{\text{min}}(t) = \left(\cos^2\phi - \sin^2\phi\right)^2$$

v resonanci $\sin^2\phi = 0$