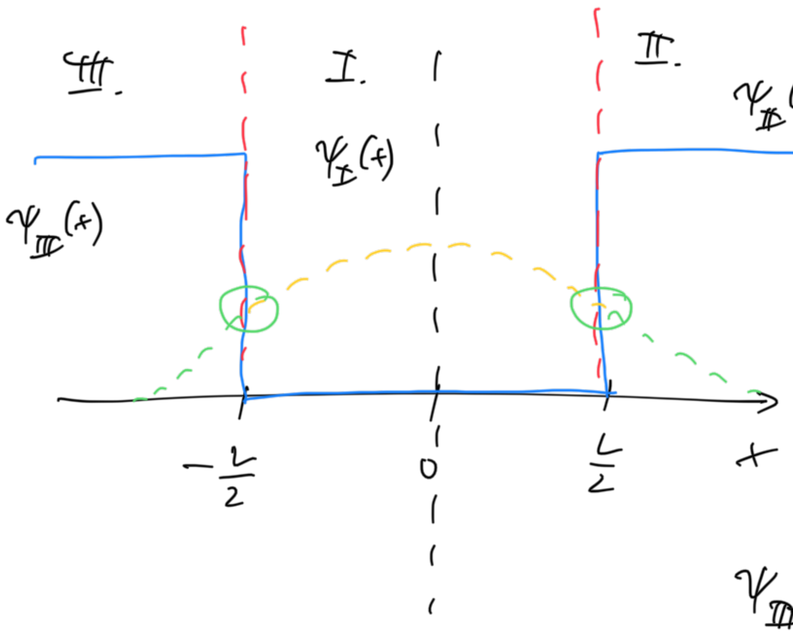
# Konečná pravouhlá potenciálna jama




$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi(x)=E\psi(x)$$

Požiadavky

$$\psi_{III}\left(-\frac{L}{2}\right)=\psi_I\left(-\frac{L}{2}\right)$$

$$\frac{\partial}{\partial x}\psi_{III}(x)\Big|_{x=-\frac{L}{2}}=\frac{\partial}{\partial x}\psi_I(x)\Big|_{x=-\frac{L}{2}}$$

$$\psi_I\left(\frac{L}{2}\right)=\psi_{II}\left(\frac{L}{2}\right)$$

$$\frac{\partial}{\partial x}\psi_I\left(\frac{L}{2}\right)\Big|_{x=\frac{L}{2}}=\frac{\partial}{\partial x}\psi_{II}(x)\Big|_{x=\frac{L}{2}}$$

$$\psi_{II}(x\to\infty)=0\ ;\ \psi_{III}(x\to-\infty)=0$$

I.

$$\psi_I(x)=A\sin(kx)+B\cos(kx)$$

reálne číslo libovoľné

II.

$x>\frac{L}{2}$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi=(E-V_0)\psi$$

$$\frac{\partial^2}{\partial x^2}\psi=-\frac{2m(V_0-E)}{\hbar^2}\psi\Rightarrow\frac{\partial^2}{\partial x^2}\psi=\xi\,\psi\Rightarrow e^{\alpha x}\ ;\ e^{-\alpha x}$$

$$\boxed{\xi=\alpha^2}$$

$$\psi_{II}(x)=C\,e^{-\alpha x}$$

III. obdobne jako II.

$$\psi_{III}(x)=D\,e^{\alpha x}$$

Symetrické řešení

$A = 0$

$C = D$

$D e^{-\alpha \frac{L}{2}} = B \cos\left(\frac{\varepsilon L}{2}\right)$ ... hodnoty funkce

$-\alpha D e^{-\alpha \frac{L}{2}} = -\varepsilon B \sin\left(\frac{\varepsilon L}{2}\right)$ ... hodnoty derivace $\Rightarrow$

$$\boxed{\alpha = \varepsilon \tan\left(\frac{\varepsilon L}{2}\right)}$$

Anti-symetrické řešení

$B = 0 \; ; \; C = -D$

$D e^{-\alpha \frac{L}{2}} = A \sin\left(\frac{\varepsilon L}{2}\right)$

$-\alpha D e^{-\alpha \frac{L}{2}} = \varepsilon A \cos\left(\frac{\varepsilon L}{2}\right) \quad \Rightarrow \quad \boxed{-\alpha = \varepsilon \cot\left(\frac{\varepsilon L}{2}\right)}$

$$\alpha^2 = \frac{2m(V_0 - E)}{\hbar^2} = \frac{2mV_0}{\hbar^2} - \frac{2m}{\hbar^2}E$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\sin(\varepsilon x) = \frac{\varepsilon^2\hbar^2}{2m}\sin(\varepsilon x) = E\sin(\varepsilon x)$$

$$E = \frac{\varepsilon^2\hbar^2}{2m}$$

$$\boxed{\alpha^2 + \varepsilon^2 = \frac{2mV_0}{\hbar^2}}$$

$u = \frac{\varkappa L}{2}$ ;

$v = \frac{\varepsilon L}{2}$ $\Rightarrow$ $\boxed{u^2 + v^2 = \frac{m V_0 L^2}{2\hbar^2}} = u_0$

sym

$\Rightarrow u = \sqrt{u_0^2 - v^2} = \frac{L\varepsilon}{2} \tan\left(\frac{L\varepsilon}{2}\right)$

anti

$= -\frac{L\varepsilon}{2} \operatorname{cotan}\left(\frac{L\varepsilon}{2}\right)$

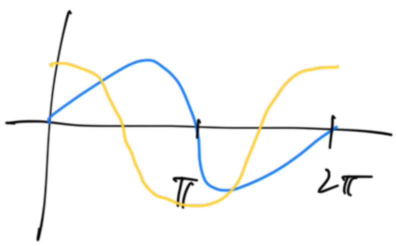
sym

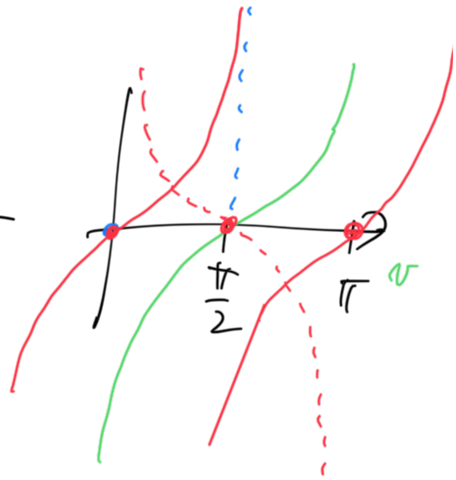
$u = v \tan v$

anti

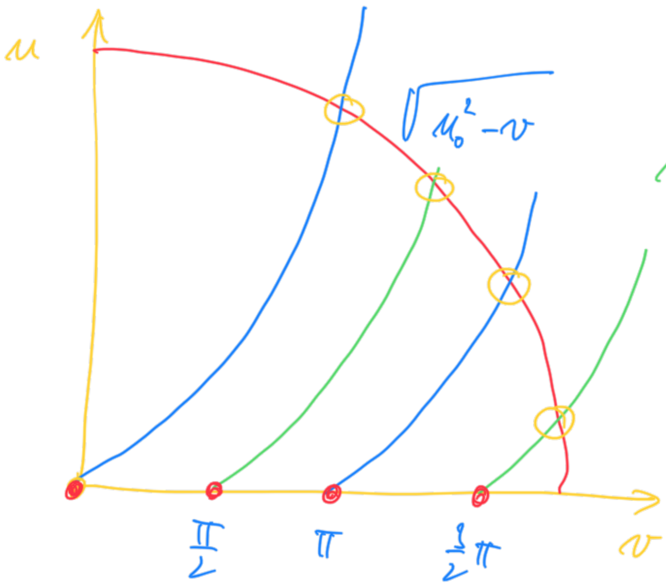
$u = -v \operatorname{cotan} v$

$\tan v = \frac{\sin v}{\cos v}$




$\operatorname{cotan} v = \frac{\cos v}{\sin v}$




$u$ a $v \Rightarrow \alpha, \varepsilon$

energie

Počet řešení! $N \Rightarrow N\frac{\pi}{2} < u_0$

$$N = \text{ground}\left(\frac{2u_0}{\pi}\right) \qquad E_n < V_0$$