Emisní čára

$$\hat{H}_I = -\hat{\vec{\mu}} \cdot \hat{\vec{E}}$$

$$\boxed{\hat{n}_{\lambda\vec{q}} = \hat{a}^{\dagger}_{\lambda\vec{q}}\, \hat{a}_{\lambda\vec{q}}}$$ počet fotonů v módu $\lambda\vec{q}$

$$n_{\lambda\vec{q}} = \frac{d}{dt} \langle \hat{a}^{\dagger}_{\lambda\vec{q}}\, a_{\lambda\vec{q}} \rangle = \langle \Psi(t) | \frac{i}{\hbar} \hat{H}\, \hat{a}^{\dagger}_{\lambda\vec{q}}\, \hat{a}_{\lambda q} | \Psi(t) \rangle$$

$$- \langle \Psi(t) | \hat{a}^{\dagger}_{\lambda q}\, a_{\lambda q} \frac{i}{\hbar} \hat{H} | \Psi(t) \rangle$$

$$\hat{H} = \hat{H}_0 + \hat{H}_L + \hat{H}_I$$

$$n_{\lambda\vec{q}} = \langle \Psi(t) | \frac{i}{\hbar} [\hat{H}_I, \hat{a}^{\dagger}_{\lambda q}\, \hat{a}_{\lambda q}] | \Psi(t) \rangle \ ; \quad \hat{H}_I = -\hat{\vec{\mu}}\, \hat{\vec{E}}$$

$$\hat{E} = i \sqrt{\frac{2\pi\hbar\omega_q}{\Omega}} \left(\hat{a}_{\lambda q} - \hat{a}^{\dagger}_{\lambda q} \right)$$

$$n_{\lambda\vec{q}} = \sqrt{\frac{2\pi\hbar\omega_q}{\hbar^2 \Omega}} \langle \Psi(t) | \hat{\mu} \left[\left(\hat{a}_{\lambda q} - \hat{a}^{\dagger}_{\lambda q} \right), \hat{a}^{\dagger}_{\lambda q}\, \hat{a}_{\lambda q} \right] | \Psi(t) \rangle$$

$$[(a - a^{\dagger}), a^{\dagger} a] = a a^{\dagger} a - a^{\dagger} a a - a^{\dagger} a^{\dagger} a + a^{\dagger} a a^{\dagger} = a + a^{\dagger}$$

$$n^{(1)}_{\lambda\vec{q}} = \sqrt{\frac{2\pi\omega_q}{\hbar\Omega}}\, Tr \left\{ \hat{\mu} \left(\hat{a}_{\lambda q} + \hat{a}^{\dagger}_{\lambda q} \right) \hat{W}^{(1)}(t) \right\}$$

1. řád pro $\hat{W}(t)$

$$\frac{\partial}{\partial t} \hat{W}(t) = -\frac{i}{\hbar} [\hat{H}, \hat{W}(t)]$$

$$\frac{\partial}{\partial t}\hat{W}^{(I)}(t) = \frac{i}{\hbar}\left[\hat{\vec{\mu}}(t)\,\hat{\vec{E}}(t),\, \hat{W}^{(I)}(t)\right]$$

$$\hat{W}^{(I)(1)}(t) = \frac{i}{\hbar}\int_0^t d\tau \left[\hat{\vec{\mu}}(\tau)\,\hat{\vec{E}}(\tau),\, \hat{W}(0)\right]$$

$$\boxed{\hat{W}^{(1)}(t) = \frac{i}{\hbar}\int_0^t d\tau\; U_S(t)\left[\hat{\vec{\mu}}(\tau)\,\hat{\vec{E}}(\tau),\, \hat{W}(0)\right]U_S^{\dagger}(t)}$$

Fluorescenční spektrum

$$f(\omega) = \underbrace{\Omega\rho(\omega)}_{\text{počet modů o frekvenci } \omega}\langle n^{(1)}_{\lambda q\sim\omega}\rangle$$

← úměrná počtu fotonů v modu o frekvenci ω

$$f(\omega) = \cancel{\Omega}\rho(\omega)\sqrt{\frac{2\pi\omega_q}{\cancel{\hbar}\cancel{\Omega}}}\,\mathrm{Tr}\Big\{\hat{\mu}\,(\hat{a}_{\lambda q}+\hat{a}^{\dagger}_{\lambda q})\,U_\perp^{\dagger}(\tau)$$

$$\times\left(\frac{i}{\hbar}\int_0^t d\tau\; U_{S+\perp}(t)\,\underbrace{U_S^{\dagger}(\tau)\,\mu\, U_S(\tau)}_{\mu(\tau)}\; i\sqrt{\frac{2\pi\cancel{\hbar}\omega_q}{\cancel{\Omega}}}\,(\hat{a}_{\lambda q}-\hat{a}^{\dagger}_{\lambda q})\,U_\perp(\tau)\,|0\rangle|e\rangle\langle e|\langle 0| \times \hat{W}_e\right) U^{\dagger}_{S+\perp}(t) - h.c.\Big\}$$

celkové znaménko je 1 $\Rightarrow$ $i\,i\,(-1) = 1$

$$= \rho(\omega_q)\frac{2\pi\omega_q}{\hbar}\,2\,\mathrm{Re}\int_0^t d\tau\;\mathrm{Tr}\Big\{\overset{d_{eg}}{\hat{\mu}}\,\hat{a}_{\lambda q}\,\underbrace{U_S(t-\tau)\,\hat{\mu}\,U_S(\tau)}_{\tilde{U}_g(t-\tau)\,d_{ge}\,\tilde{U}_e(\tau)\,e^{-\frac{i}{\hbar}\varepsilon_e\tau}}|e\rangle\langle e|\hat{W}_e\,\underbrace{U_S^{\dagger}(t)}_{\tilde{U}_e^{\dagger}(t)\,e^{\frac{i}{\hbar}\varepsilon_e t}}$$

$\tilde{U}_g(t-\tau) = e^{-\frac{i}{\hbar}\varepsilon_g(t-\tau)}$

$$\times\,\underbrace{U_\perp(t-\tau)\,\hat{a}^{\dagger}_{\lambda q}\,U_\perp(\tau)}_{e^{-i\omega_q(t-\tau)}}\,|0\rangle\langle 0|\,U_\perp^{\dagger}(t)\Big\}$$

$$\frac{|d_{eg}|^2}{3}$$

$$= \rho(\omega_q)\frac{2\pi\omega_q}{\hbar}\,2\,\mathrm{Re}\int_0^t d\tau\; e^{-i\omega_q(t-\tau)}\,\langle|d_{eg}|^2\rangle\, e^{-\frac{i}{\hbar}(\varepsilon_g-\varepsilon_e)(t-\tau)}$$

$$\times\, Tr_B\left\{\tilde{V}_g(t-\tau)\,\tilde{V}_e^{\dagger}(t-\tau)\,w_{eq}^{(e)}\right\}$$

$$= \rho(\omega_q)\,\frac{2\pi\omega_q}{\hbar}\,\frac{|d_{eg}|^2}{3}\,2\,\mathrm{Re}\int_0^\infty d\tau\, e^{i(\omega_{eg}-\omega_q)\tau}\,\underbrace{Tr_B\left\{\tilde{V}_g(\tau)\tilde{V}_e^{\dagger}(\tau)\,w_{eq}^{(e)}\right\}}_{\rho_{ge}(\tau)\,=\,\rho_{ge}(0)\,e^{-\gamma\tau}}$$

$$\rho(\omega_q) = \frac{\omega_q^3}{c^3\pi^2}$$

$$f(\omega) = \frac{2\omega_q^3}{3\hbar c^3\pi}\,|d_{eg}|^2\,\underbrace{2\,\mathrm{Re}\int_0^\infty d\tau\, e^{i(\omega_{eg}-\omega_q)\tau}\,e^{-\gamma\tau}}_{G_f(\omega)}$$

$$G_f(\omega) \approx \frac{\gamma}{\gamma^2+(\omega_{eg}-\omega_q)^2}$$

$$\boxed{f(\omega) = \frac{2\omega^3}{3\hbar c^3\pi}\,|d_{eg}|^2\,G_f(\omega)}$$

$$\gamma\to 0 \qquad G_f(\omega) = \delta(\omega_{eg}-\omega_q)$$

Vztah mezi spektrem a dobou života

$$|e\rangle|0\rangle \longrightarrow |g\rangle|1\rangle$$

$$\frac{\partial}{\partial t}P_{g1}^{\lambda q} = K(\omega_q)\,P_{e0}^{\lambda q}$$

$$K(\omega_q) = Tr\left\{\hat{n}_q\; W^{(1)}(t)\right\} = \frac{f(\omega)}{\Omega\,\rho(\omega)}$$

Pravděpodobnost obsazení stavu $|g\rangle$

$$P_g = \sum_{\lambda q} P_{g1}^{\lambda q}$$

$$\frac{\partial}{\partial t} P_g = \underbrace{\sum_{\lambda q} K(\omega_q)}_{\frac{1}{\tau_e}} P_e^{\equiv 1} = \Omega \int_0^\infty d\omega\, \rho(\omega) K(\omega) P_e^{\equiv 1}$$

$$\frac{1}{\tau_e} = \Omega \int_0^\infty d\omega\, \rho(\omega) K(\omega) = \int_0^\infty d\omega\, f(\omega)$$

$$\boxed{\frac{1}{\tau_e} = \int_0^\infty d\omega \frac{2}{3\hbar c^3 \pi} |d_{eg}|^2 \omega^3 G_f(\omega)}$$

$G_f \leftarrow \delta(\omega - \omega_{eg})$

$$\boxed{= \frac{2\omega_{eg}^3 |d_{eg}|^2}{3\hbar c^3 \pi}}$$